TRIBHUVAN UNIVERSITY

INSTITUTE OF ENGINEERING

Examination Control Division

(Sample Question)

Exam.	**Regular (New Course)**		
Level	BE	**Full Marks**	60
Programme	All Except BAR	**Pass Marks**	24
Year / Part	I / II	**Time**	3 hrs.

Subject: - Engineering Mathematics II (SH 151)

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt* ***All*** *questions.*
- ✓ *The figures in the margin indicate* ***Full Marks***.
- ✓ *Assume suitable data if necessary.*

1 (a) $\log(x^3 + y^3 - x^2y - xy^2)$, then show that $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y}\right)^2 u = -\frac{4}{(x+y)^2}$ [2]

(b) If $u = \sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, then show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\tan u$ [2]

2 (a) Evaluate $\int_0^{\pi}\int_0^{x} \sin y \; dx\, dy$ [2]

(b) Evaluate $\iiint_y^x xyz\, dx\, dy\, dz$ over the sphere $x^2 + y^2 + z^2 = a^2$ in first octant [2]

3 (a) A particle moves along the curve $x = 4\cos t,\ y = 4\sin t,\ z = 6t$, then find the velocity and acceleration at time $t = \frac{\pi}{2}$ [2]

(b) Find the unit normal vector to the surface $xy^3z^2 = 4$, at the point $(-1,-1,2)$ [2]

(c) If $\phi = x^3 + y^3 + z^3 - 3xyz$, then show that *curl (grad* $\phi) = 0$ [2]

4 (a) Find the Laplace Transform of the function: $\frac{\sin^2 t}{t}$ [2]

(b) Find the inverse Laplace transform of $\frac{s^2+s-2}{s(s+3)(s-2)}$ [2]

5 (a) Find the rank of the following matrix: [2]

$$\begin{bmatrix} 1 & 2 & 0 & -1 \\ 3 & 4 & 1 & 2 \\ -2 & 3 & 2 & 5 \end{bmatrix}$$

(b) Test whether the vectors (1,2,-1),(1,2,4) and (3,0,1) are linearly independent or dependent. [2]

6 Solve $y^2 + y=0$, by power series method. [2]

7 Find the minimum value using Lagrange multiplier method of $x^2 + y^2 + z^2$ subject to the condition $ax + by + cz = p$. [4]

8 Change the order of integration and evaluate $\int_0^1 \int_x^{\sqrt{2x-x^2}} \frac{x}{\sqrt{(x^2+y^2)}}\, dxdy$ [4]

9 Prove that "A line integral $\int_C \vec{F} \cdot d\vec{r}$ is independent of path C joining any two points A and B if and only if $\vec{F} = \nabla\phi$ for some scalar function ϕ" [4]

10 Using Green's theorem, evaluate the line integral $\int_C (3x^2 - 8y^2)dx + (4y - 6xy)dy$ where C is the boundary of the region $y = \sqrt{x}$ and $y = x^2$ [4]

11 Using Gauss divergence theorem to evaluate the surface integral $\int\int_S \vec{F}.\vec{n}\, ds$ for [4]

$\vec{F} = xy\,\vec{i} - xz^2\vec{j} + yz\vec{k}$ where S is the surfaces $x + y + z = 1, x = 0, y = 0, z = 0$

12 Using the Laplace transform technique, solve the initial value problem: [4]

$$y''(t) + 4y'(t) + 3y(t) = e^{-t}, \;\; y(0) = 0, y'(0) = 1$$

13 Find the eigen values and eigen vectors of the Matrix $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ [4]

14 Reduce the quadratic form $Q(x) = 2x_1^2 + x_2^2 + 2x_3^2 + 2x_1x_2 + 2x_2x_3 + 2\,x_1x_3$ into canonical form. [4]

15 Show that $J_{-\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}}\cos x$, where $J_{-\frac{1}{2}}(x)$ is Bessel's function. [4]

OR

Show that $n\,P_n(x) = x\,P_n'(x) - n\,P_{n-1}'(x)$, where $P_n(x)$ is Legendre's polynomial. [4]

TRIBHUVAN UNIVERSITY INSTITUTE OF ENGINEERING **Examination Control Division** **2081 Ashwin**	**Exam.**	**Regular (New Course -2080 Batch)**		
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	Programme	All(Except BAR)	**Pass Marks**	24
	Year / Part	I / II	**Time**	3 hrs.

Subject: - Engineering Mathematics II *(SH 151)*

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1. a) By using Euler's theorem show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$
   where, $u = \tan^{-1}(x^2 + 2y^2)$ [2]

   b) If $u = x^2$ and $v = y^2$, then find $\frac{\partial(u,v)}{\partial(x,y)}$ [2]

2. a) Evaluate $\int_0^1 \int_0^z \int_{x-z}^{x+z} (x + y + z)\, dy\, dx\, dz$ [2]

   b) Evaluate the mass of the solid region bounded by $z = 1 - x^2$ and planes $z = 0$, $y = 1, y = -1$ with $\rho(x, y, z) = z(y + 2)$ [2]

3. a) Find the directional derivative of $\Phi(x, y) = 4x^2 + 3y - 4z$ at $(1,\ 2, 1)$ in the direction of $2\vec{i} + 2\vec{j} + \vec{k}$ [2]

   b) A particle moves along the curve $x = \sqrt{2}\cos t,\ y = \sqrt{2}\sin t,\ z = 4t$, then find the velocity and acceleration at time $t = \frac{\pi}{4}$ [2]

   c) If $\phi = x^3 + y^3 + z^3 - 3xyz$, then find *div (grad* ϕ*)* at the point $(1, -1,\ 1)$ [2]

4. a) Find the Laplace Transform of the function: $\frac{1-e^t}{t}$ [2]

   b) Find the inverse Laplace transform of $\frac{s}{(s+2)^3}$ [2]

5. a) Find the rank of the following matrix:
$$\begin{bmatrix} 2 & -4 & 3 & 1 & 0 \\ 1 & -2 & 1 & -4 & 2 \\ 0 & 1 & -1 & 3 & 1 \end{bmatrix}$$
[2]

   b) Test whether the vectors $(1, 1, 1), (1, -1,\ 1)$ and $(2, 0, 3)$ are linearly independent or dependent. [2]

6. Express $2x^2 - 4x + 2$ as the Legendre's polynomials. [2]

7. Find the minimum value using Lagrange multiplier method of $x^2 + xy + y^2 + 3z^2$ subject to the condition $x + 2y + 4z - 60 = 0$. [4]

8. Change the order of integration and evaluate $\int_0^a \int_0^x \frac{\cos y}{\sqrt{(a-x)(a-y)}} dx dy$. [4]

9. Prove that the necessary and sufficient conditions for a vector function $\vec{a}$ of a scalar variable t to have a constant direction is $\vec{a} \times \frac{d\vec{a}}{dt} = 0$ [4]

10. Find the area of asteroid $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{2/3}$ using Green's theorem. [4]

11. Apply Gauss-Divergence theorem to evaluate $\int\int_s \vec{F}.\vec{n}ds$ for $\vec{F} = x\vec{i} - y\vec{j} + (z^2 - 1)\vec{k}$ and s is the cylinder formed by the surface $z = 0,\ z = 1$, and $x^2 + y^2 = 4$ [4]

12. Using the Laplace transform technique, solve the initial value problem: [4]

$$y''(t) - y'(t) + 6y(t) = e^{-t},\ y(0) = 0, y'(0) = 0$$

13. Find the eigen values and eigen vectors of the matrix $\begin{bmatrix} 3 & 2 & 2 \\ 1 & 4 & 1 \\ -2 & -4 & -1 \end{bmatrix}$ [4]

14. Reduce the quadratic form $Q(x) = 6x_1^2 + 3x_2^2 + 3x_3^2 - 4x_1x_2 - 2x_2x_3 + 4\,x_1x_3$ into canonical form. [4]

15. Solve $y'' - 4xy' + (4x^2 - 2)y = 0$ by power series method. [4]

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TRIBHUVAN UNIVERSITY	Exam.	Back (New Course)		
INSTITUTE OF ENGINEERING	Level	BE	Full Marks	60
Examination Control Division	Programme	All (Except BAR)	Pass Marks	24
2082 Baishakh	Year / Part	I / II	Time	3 hrs.

Subject: - Engineering Mathematics II *(SH 151)*

- ✓ Candidates are required to give their answers in their own words as far as practicable.
- ✓ *Attempt **All** questions.*
- ✓ *The figures in the margin indicate **Full Marks**.*
- ✓ *Assume suitable data if necessary.*

1. a) If $u = \log(x^2 + y^2 + z^3 - 3xyz)$, then show that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = \frac{3}{x+y+z}$ [2]

   b) If $u = \tan^{-1}(x^2 + 2y^2)$, then show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$ [2]

2. a) Evaluate $\int_1^2 \int_0^x \frac{1}{x^2 + y^2} dxdy$ [2]

   b) Find the volume of the hemisphere $x^2 + y^2 + z^2 = a^2$ using triple integral. [2]

3. a) A particle move along the curve $x = 2t^2$, $y = t^2 - 4t$, $z = 3t - 5$, then find the components of the velocity in the direction of $\vec{a} = \vec{i} - 3\vec{j} + 2\vec{k}$ at $t = 1$ [2]

   b) Find the angle between two surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at $(2, -1, 2)$ [2]

   c) If $\vec{v} = x^2yz\vec{i} + xy^2z\vec{j} + xyz^2\vec{k}$, then show that $div\ (curl\ \vec{v}) = 0$ [2]

4. a) Find the Laplace transform of $t^2 \cos at$ [2]

   b) Find the inverse Laplace transform of $\frac{1}{4s + s^3}$ [2]

5. a) Find the rank of the following matrix [2]

$$\begin{bmatrix} 2 & 4 & -4 & 3 \\ 1 & -2 & -1 & 1 \\ 1 & 2 & -1 & 3 \end{bmatrix}$$

   b) Test whether the vectors $(3, -1, 4)$, $(2, 2, -3)$ and $(0, -4, 1)$ are linearly independent. [2]

6. Solve $y'' = 2y$, by power series method. [2]

7. Find the minimum value of $x^2 + y^2 + z^2$ using Lagrange multiplier to the condition $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$ [4]

8. Find the moment of inertia of the region of the ellipse in first quadrant with mass M and density proportional $\rho(x, y) = k\,xy$ about x-axis. [4]

9. Prove that the necessary and sufficient conditions for a vector function $\vec{a}$ of a scalar variable t to have a constant magnitude is $\vec{a} \cdot \frac{d\vec{a}}{dt} = 0$ [4]

   OR

   Show that the line integral $\int_C \vec{F} \cdot d\vec{r}$ is independent of path for $\vec{F} = (y \sin z - \sin x)\vec{i} + (x \sin z + 2yz)\vec{j} + (xy \cos z + y^2)\vec{k}$, and find its scalar potential.

10. Find the flux of $\vec{F} = x^2\vec{i} - y^2\vec{j} + z^2\vec{k}$ over the plane surface $x + y + z = 1$ lying in the first octant. [4]

11. Apply Gauss-Divergence theorem to evaluate $\int\int_S \vec{F}.\vec{n}ds$ for $\vec{F} = 4\,xz\,\vec{i} - y^2\vec{j} + y\,z\,\vec{k}$ over the surface of the cube $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$. [4]

12. Using the Laplace transform technique, solve the initial value problem: [4]

$$y''(t) + 2y'(t) + 5y(t) = e^{-t}\sin t\,,\ \ y(0) = 1, y'(0) = 1$$

13. Find the eigen values and eigen vectors of the matrix $\begin{bmatrix} 4 & 2 & -1 \\ -5 & 3 & 2 \\ -2 & 4 & 1 \end{bmatrix}$ [4]

14. Reduce the quadratic form $Q(x) = 2x_1^2 + x_2^2 + x_3^2 + 2x_1x_2 - 2x_1x_3 + 4\,x_2x_3$ into canonical form. [4]

15. Show that $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}}\sin x$, where $J_{\frac{1}{2}}(x)$ is Bessel's function. [4]

OR

Show that $(n+1)P_{n+1}(x) = (2n+1)x\,P_n(x) - n\,P_{n-1}(x)$, where $P_n(x)$ is the Legendre's polynomial.

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